

Tangent Planes and Linear Approximation

1. Let S be the cylinder $x^2 + y^2 = 4$. Find the plane tangent to S at the point $(1, \sqrt{3}, 5)$.

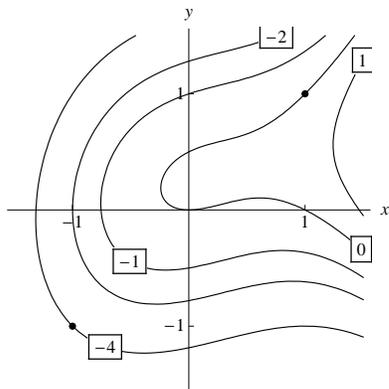
2. Let S be the surface $z = y \sin x$. Find the plane tangent to S at the point $(\frac{\pi}{6}, 2, 1)$.

3. Let S be the graph of $f(x, y)$; that is, S is the surface $z = f(x, y)$. Find the plane tangent to S at the point $(a, b, f(a, b))$.

(Notice that #2 was a special case of this, with $f(x, y) = y \sin x$, $a = \frac{\pi}{6}$, and $b = 2$.)

4. Let $f(x, y, z) = \sqrt{x} + xyz$. Use linear approximation to approximate the value of $f(1.1, 1.9, 3.1)$.

5. Suppose the mysterious function $f(x, y)$ has the following level set diagram (contour map).



The points $(1, 1)$ and $(-1, -1)$ are marked with dots. Let $L_1(x, y)$ be the linearization of f at $(1, 1)$ and $L_2(x, y)$ be the linearization of f at $(-1, -1)$. Which of the following is the level set diagram of $L_1(x, y)$? Which of the following is the level set diagram of $L_2(x, y)$?

